

Supplementary Homework Resources

Numeracy Progressions



Supplementary Homework Resources Numeracy Progressions - ADDITIVE STRATEGIES -

Numeracy is fundamental to a student's ability to learn at school and to engage productively in society. Sudents become numerate as they develop the knowledge and skills to use mathematics confidently across learning areas at school and in their lives more broadly

How to use this resource

This document contains a series of progressions and examples that you could use with your child when discussing mathematics. These progressions are observable indicators and behaviours your child may demonstrate as they develop their mathematical knowledge and skills. These Numeracy Progressions are based on the National Numeracy Progressions / Australian Curriculum Version 9 Numeracy General Capabilities.



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Numeracy: Additive Strategies - Progression 1

Your child may be demonstrating these behaviours:

Emergent strategies

- describes the effects of "adding to" and "taking away from" a collection of objects
- combines 2 groups of objects and attempts to determine the total

Numeracy: Additive Strategies - Progression 2 Your child may be demonstrating these behaviours: Example:

Perceptual strategies		
•	represents additive situations involving a small number of items with objects, drawings and diagrams	
•	counts or subitises all items to determine the result when 2 collections are combined or when a quantity is taken away from a collection	When told <i>"I have 3 red bottle tops in this pile and 2 blue bottle tops in this pile; how many do I have altogether?"</i> student counts each bottle top <i>"one, 2, 3"</i> then <i>"4, 5",</i> responding <i>"5"</i>
•	changes a quantity by adding to or taking from an initial quantity, using physical or virtual materials or fingers	
•	combines 2 or more objects to form collections up to 10 and partitions collections of up to 10 items, to identify smaller quantities within the collection	

Numeracy: Additive Strategies - Progression 3		
Your child may be demonstrating these behaviours:	Example:	
 Figurative solves additive tasks involving 2 concealed collections of items by visualising the numbers, then counts from one to determine the total 	Constructs a mental image of 5 and of 3 but when asked to combine to give a total, counts from one and may use head gestures to keep track of the count	



Numeracy: Additive Strategies - Progression 4

Your child may be demonstrating	these behaviours:	Example:
Counting on (by ones) represents and uses a range addition problems such as from 	ge of counting strategies to solve counting-up-to and counting-up-	To solve <i>"I have 7 apples. I want 10. How many more do I need?"</i> , your child counts the number of apples needed to increase the quantity from 7 to 10 Uses a counting on strategy to calculate 6 + 3, says <i>"6, 7, 8 o it</i> 's 9"
		To solve 6 + ? = 9, your child says <i>"6 7, 8, 9 it's 3"</i>
uses the additive property change in value when zero	of zero, that a number will not is added to or taken away from it	When asked what is 5 + 0, your child responds <i>"5"</i>

Numeracy: Additive Strategies - Progression 5		
Your child may be demonstrating these behaviours:	Example:	
Counting back (by ones)		
 represents and uses a range of counting strategies to solve subtraction problems such as counting-down-from, counting up from, counting-down-to 	To solve <i>"Mia had 10 cupcakes. She gave 3 cupcakes away. How many cupcakes does Mia have left?"</i> your child counts back from 10, <i>"9, 8, 7, Mia has 7 left"</i>	
	To solve 12 take away something equals 8, your child responds <i>"12 take away one is 11, then 10, 9, 8 It's 4"</i>	



Numeracy: Additive Strategies - Progression 6			
Your child may be demonstrating these behaviours:	Example:		
Flexible strategies with combinations to 10			
 describes subtraction as the difference between numbers rather than taking away using diagrams and a range of representations 	Using a number line to represent 8 – 3 as the difference between 8 and 3		
 uses a range of strategies to add or subtract 2 or more 	Bridging to 10		
numbers within the range of 1–20	Near doubles		
	Adding the same to both numbers 7 + 8 = 15 because double 8 is 16, and 7 is one less than 8		
	8 + 6 = 14 because 8 + 2 = 10 and 4 more is 14		
	15 – 8 = 7 because I can add 2 to both to give 17 – 10 = 7		
 uses knowledge of part-part-whole number construction to partition natural numbers into parts to solve addition and subtraction problems 	To solve 6 + ? = 13, your child says "6 plus 4 makes 10, and 3 more so it's 7"		
 represents additive situations using number sentences and part-part-whole diagrams including when different parts or the whole are unknown 	Uses the number sentence 8 – 3 = 5 to represent the problem <i>"I had 8 pencils. I gave 3 to Max. I now have 5 remaining"</i>		
	Matches the number sentence 4 + ? = 9 to the problem, <i>"I have 9 cups and only 4 saucers, how many more saucers do I need?"</i>		



Numeracy: Additive Strategies - Progression 7			
Your child may be demonstrating these behaviours:	Example:		
 Flexible strategies with two-digit numbers chooses from a range of known strategies to solve additive problems involving two-digit numbers 	Uses place value knowledge, known addition facts and part-part-whole number knowledge to solve problems like: 24 + 8 + 13 partitions 8 as 6 and 2 more combines 24 and 6 to rename it as 30 combines it with 13 to make 43 combines the remaining 2 to find 45 Adds the same quantity to both numbers: 47 - 38 = 49 - 40		
 identifies that the same combinations and partitions to 10 are repeated within each decade 	Knowing that 8 + 2 = 10, your child knows 18 + 2 = 20 and 28 + 2 = 30 etc.		
 identifies addition as associative and commutative, and that subtraction is neither 			
 applies the commutative and associative properties of addition to simplify mental computation 	To calculate 23 + 9 + 7, your child adds 23 to 7 to get 30, then adds 9 to give 39		
 applies inverse relationship of addition and subtraction to solve problems, including solving problems with digital tools, and uses the inverse relationship to justify an answer 	When solving $23 - 16$, your child chooses to use addition such as $16 + ? = 23$ When using a calculator to solve $16 + ? = 38$, your child decides to use subtraction and inputs $38 - 16$		
 represents a wide range of additive problem situations involving two-digit numbers using appropriate addition and subtraction number sentences 			



Numeracy: Additive Strategies - Progression 8		
Your child may be demonstrating these behaviours:	Example:	
 Flexible strategies with three-digit numbers and beyond uses place value, standard and non-standard partitioning, trading or exchanging of units to mentally add and subtract numbers with 3 or more digits 	 To add 250 and 457, your child: partitions 250 into 2 hundreds and 5 tens says 457 plus 2 hundreds is 657, plus 5 tens is 707 To add 184 and 270, your child: partitions into 150 + 34 + 250 + 20 = 400 + 34 + 20 = 454 	
 chooses and uses strategies including algorithms and technology to efficiently solve additive problems 	Develops total costings for ingredients or materials for a task, or combines measurements to determine the total amount of materials required	
 uses estimation to determine the reasonableness of the solution to an additive problem 	When asked to add 249 and 437, your child says <i>"250</i> + <i>440 is 690"</i>	
 represents a wide range of familiar real-world additive situations involving large numbers as standard number sentences, explaining their reasoning 		



Numeracy: Additive Strategies - Progression 9

Your child may be demonstrating these behaviours:

Flexible strategies with fractions and decimals

- uses knowledge of place value and how to partition numbers in different ways to make the calculation easier when adding and subtracting decimals with up to 3 decimal places
- identifies and justifies the need for a common denominator when solving additive problems involving fractions with related denominators
- represents a wide range of familiar real-world additive situations involving decimals and common fractions as standard number sentences, explaining their reasoning

Numeracy: Additive Strategies - Progression 10

Your child may be demonstrating these behaviours:

Flexible strategies with rational numbers

- uses knowledge of equivalent fractions, multiplicative thinking and how to partition fractional numbers to make calculations easier when adding and subtracting fractions with different denominators
- solves additive problems involving the addition and subtraction of rational numbers, including fractions with unrelated denominators and integers
- chooses and uses appropriate strategies to solve multi-step problems involving the addition and subtraction of rational numbers



Written methods and the connection to mental computation strategies

It is critical that students develop trusted, efficient and flexible methods for adding and subtracting whole numbers and decimals. The development of trusted, compact and efficient written methods occurs alongside students' use of personal mental strategies. Written methods commence with informal drawings and jottings that include diagrams, words and numerals. They increase in formality and abstraction as students progress through their primary years of schooling.

Our goal is to develop in every child, **trusted**, **compact** and **efficient** written methods for calculating in the four operations with whole numbers and decimals. For most students, it is anticipated that this will occur by the end of Year 6.

A central and guiding principle relating to written methods is that *the presentation and rehearsed application of a standard written algorithm for addition and subtraction should be delayed*. The delayed appearance of the standard written algorithm allows time for students to develop confidence and fluency in the application of personal mental methods. Students who can confidently and meaningfully apply mental computation strategies to solve addition and subtraction problems prove to be more able to understand and apply the standard algorithm.

Key messages

Our goal for students is that they develop **trusted**, **compact** and **efficient** written methods for calculating in the four operations with whole numbers and decimals. In achieving this, there are several key messages:

- Informal written methods should reflect students' personal mental strategies for adding and subtracting numbers.
- Informal written methods can include drawings, diagrams, words and symbols.
- Students' personal methods will often develop in a unique sequence.
- Students' informal written methods will vary greatly. They will be influenced by a students' age, the context for the computation, the size and nature of the numbers and the mental computation strategy used.
- Students should be regularly encouraged to share their personal strategies and written methods with other students.
 Students' efficient use of mental computation strategies remains the initial priority.
- Fluency with standard algorithms becomes crucial as students work with larger numbers and mental strategies become less efficient.



Everton Park State School NUMERACY PROGRESSIONS







Increasing formality

ncreasing formality

> Year 6



Supplementary Homework Resources Numeracy Progressions - MULTIPLICATIVE STRATEGIES -

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Numeracy: Multiplicative Strategies - Progression 1

Your child may be demonstrating these behaviours:	Example:
Forming equal groupsshares collections equally by dealing	Your child distributes all items one-to-one until they are exhausted, checking that the final groups are equal
 makes equal groups and counts by ones to determine the total 	

Numeracy: Multiplicative Strategies - Progression 2		
Your child may be demonstrating these behaviours:		Example:
Perceptu •	ual multiples uses groups or multiples in counting and sharing physical or virtual materials	Skip counts by twos, fives or tens with all objects visible
•	represents authentic situations involving equal sharing and equal grouping with drawings and physical or virtual materials	Your child draws a picture to represent 4 tables that seat 6 people to determine how many chairs they will need; Your child uses 8 counters to represent sharing \$8 between 4 friends

Numeracy: Multiplicative Strategies - Progression 3

Your child may be demonstrating these behaviours:	Example:
Figurative uses perceptual markers to represent concealed quantities of equal amounts to determine the total number of items 	To count how many whiteboard markers are in 4 packs, and knowing they come in packs of 5, your child counts the number of markers as 5, 10, 15, 20



Numeracy: Multiplicative Strategies - Progression 4

Your child may be demonstrating these behaviours:	Example:
 Repeated abstract composite units uses composite units in repeated addition using the unit a specified number of times 	Your child interprets "4 lots of 3" additively and calculates 3 + 3 + 3 + 3 answering <i>"12"</i>
 uses composite units in repeated subtraction using the unit a specified number of times 	When asked "how many groups of 4 can be formed from our class of 24?", your child repeatedly takes away 4 from 24 and counts the number of times this can be done. Your child says "20, 16, 12, 8, 4 and zero so we can form 6 groups of 4"

Numeracy: Multiplicative Strategies - Progression 5		
Your child may be demonstrating these behaviours:	Example:	
 Coordinating composite units identifies and represents multiplication in various ways and solves simple multiplicative problems using these representations 	Your child represents multiplication as equal groups and arrays	
 identifies and represents division in various ways such as sharing division or grouping division 	To share a carton of 12 eggs equally between 4 people, your child draws 12 dots and circles 3 groups of 4 with 3 in each share	
 identifies and represents multiplication and division abstractly using the symbols × and ÷ 	Your child represents 3 groups of 4 as 3 × 4; uses 9 ÷ 3 to represent 9 pieces of fruit being equally shared by 3 people	



Numeracy: Multiplicative Strategies - Progression 6		
Your child may be demonstrating these behaviours:	Example:	
 Flexible strategies for single-digit multiplication and division draws on the structure of multiplication to use known multiples in calculating related multiples 	Your child uses multiples of 4 to calculate multiples of 8	
 interprets a range of multiplicative situations using the context of the problem to form a number sentence 	To calculate the total number of buttons in 2 containers, each with 5 buttons, your child uses the number sentence $2 \times 5 = ?$ If a packet of 20 pens is to be shared equally between 4, your child writes $20 \div 4 = ?$	
 demonstrates flexibility in the use of single-digit multiplication facts 	7 boxes of 6 donuts is 42 donuts altogether because 7 × 6 = 42 Multiplying any factor by one will always give a product of that factor i.e. 1 × 6 = 6 If you multiply any number by zero the result will always be zero	
 uses the commutative and distributive properties of multiplication to aid computation when solving problems 	5 × 6 is the same as 6 × 5 calculates 7 × 4 by adding 5 × 4 and 2 × 4	
 applies mental strategies for multiplication to division and can justify their use 	To divide 64 by 4, your child halves 64 then halves 32 to get an answer of 16	
 explains the idea of a remainder as what is "left over" from the division 	An incomplete group, lot of, next row or multiple	



Numeracy: Multiplicative Strategies - Progression 7

Your child may be demonstrating these behaviours:	Example:
 Flexible strategies for multiplication and division uses multiplication and division as inverse operations to solve problems including solving problems with digital tools and to justify a solution 	When solving 14 × ? = 336, your child chooses to use division 336 ÷ 14 = ? Your child determines how long it will take to save up for a purchase and tests the effect of changing the amount saved each period
 uses known mental and written strategies, such as using the distributive property, partitioning into place value or factors to solve multiplicative problems involving numbers with up to 3 digits, and can justify their use 	7 × 83 equals 7 × 80 plus 7 × 3 To multiply a number by 48, your child first multiplies by 12 and then multiplies the result by 4 To solve 16 × 15, your child uses double and half, such as 16 × 15 = 8 × 30
 uses estimation and rounding to check the reasonableness of products and quotients 	Your child multiplies 200 by 30 to determine if 6138 is a reasonable answer to 198 × 31

Numeracy: Multiplicative Strategies - Progression 8

Your ch	ild may be demonstrating these behaviours:	Example:
Flexible •	strategies for multi-digit multiplication and division solves multi-step problems involving multiplicative situations using appropriate mental strategies, digital tools and algorithms	Your child uses a rate of application to determine the amount of paint required to cover a large area and determines how many tins of paint are required
•	interprets, represents and solves multifaceted problems involving all 4 operations with natural numbers	



Numeracy: Multiplicative Strategies - Progression 9			
Your child may be demonstrating these behaviours:	Example:		
Flexible strategies for multiplication and division of rational numbers			
 expresses a number as a product of its prime factors for a purpose 			
 expresses repeated factors of the same number in exponent form 	$2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$		
 identifies and describes products of the same number as square or cube numbers 	3×3 is the same as 3^2 which is read as 3 squared		
 describes the effect of multiplication by a decimal or fraction less than one 	When multiplying natural numbers by a fraction or decimal less than one such as $15 \times \frac{1}{2} = 7.5$		
 connects and converts decimals to fractions to assist in mental computation involving multiplication or division 	To calculate 16 × 0.25, your child recognises 0.25 as a quarter, and determines a quarter of 16 or determines 0.5 ÷ 0.25, by reading this as "one half, how many quarters?" and gives the answer as 2		
 calculates the percentage of a quantity flexibly using multiplication and division 	To calculate 13% of 1600, your child uses 0.13 × 1600 or 1600 ÷ 100 × 13		
 uses multiplicative strategies efficiently to solve problems involving rational numbers including integers 	Calculates the average temperature for Mt Wellington for July to be -1.6 °C		



Numeracy: Multiplicative Strategies - Progression 10		
Your child may be demonstrating these behaviours:	Example:	
 Flexible strategies for working multiplicatively uses knowledge of place value and multiplicative partitioning to multiply and divide decimals efficiently 	0.461 × 200 = 0.461 × 100 × 2 = 46.1 × 2 = 92.2	
 flexibly operates multiplicatively with extremely large or very small numbers expressed in scientific notation 	Your child calculates the area of a computer chip measuring 2.56 × 10 ⁻⁶ m in width by 1.4 × 10 ⁻⁷ m in length	
 chooses and uses appropriate strategies to solve multi-step problems and model situations involving rational numbers 		
 represents and solves multifaceted problems in a wide range of multiplicative situations including scientific notation for those involving very small or very large numbers 	Your child chooses to calculate the percentage of a percentage to determine successive discounts Your child determines the time it takes for sunlight to reach the earth	



Written methods and the connection to mental computation strategies

It is critical that students develop trusted, efficient and flexible methods for multiplying and dividing whole numbers and decimals. The development of trusted, compact and efficient written methods occurs alongside student use of personal mental strategies.

Written methods commence with informal drawings and jottings that include diagrams, words and numerals. They increase in formality and abstraction as students progress through their primary years of schooling. A standard written algorithm is valuable when the context of the operation is complicated or the size of the numbers inhibits the efficient use of a mental computation strategy.

Our goal is to develop in every child, **trusted**, **compact** and **efficient** written methods for calculating in the four operations with whole numbers and decimals. For most students, it is anticipated that this will occur by the end of Year 6.

A central and guiding principle relating to written methods is that **the presentation and rehearsed application of a standard written algorithm for multiplication and division should be delayed**. The delayed appearance of the standard written algorithm allows time for students to develop confidence and fluency with personal mental methods. Students who confidently and meaningfully apply mental computation strategies are better equipped to solve everyday problems, including those that require the use of a standard algorithm.

Key messages

To develop **trusted**, **compact** and **efficient** written methods, there are several key messages for teaching and learning:

- informal written methods reflect students' personal mental strategies for multiplying and dividing numbers
- informal written methods can include drawings, diagrams, words and symbols
- students' personal methods will often develop in a unique sequence
- students' informal written methods will vary greatly. They will be influenced by a students' age, the context for the computation, the size and nature of the numbers and the mental computation strategy used
- students' personal written methods are developed and enhanced through opportunities to share, evaluate and trial the methods of other students.





> Year 6

→ Year 6



Increasing formality

ncreasing formality



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Numeracy – Glossary

addition facts

The results associated with the sums of pairs of natural numbers from 0 to 9. They are foundational to arithmetic.

additive

A situation or relationship that involves addition, subtraction or both, e.g. giving change from a simple money transaction.

approximate

To obtain or state a value to a particular accuracy.

approximation

A result which is not exact, but is close enough for a given purpose, e.g. giving an approximation of the area of a complex shape by using a combination of basic shapes.

associative

Of or relating to an operation that when applied to any 3 elements of an expression is the same regardless of which pair of elements (without changing their order) is combined first.

base-10

A number system which uses the digits 0–9 and the value of the digit is determined by its face value and its place value, e.g. $283 = 2 \times 100 + 8 \times 10 + 3 \times 1$ and 283 = 200 + 80 + 3.

commutative property

In general, the commutative property of addition and multiplication of real numbers is that for all real numbers a and b, a + b = b + a and $a \times b = b \times a$ respectively.

conceptually subitise

The ability to recognise a whole quantity as the result of recognising smaller quantities, e.g. 5 can be seen as 3 and 2 or 4 and 1.

counting

The process of quantifying the number of objects in a set or collection.

counting down strategy

To answer a question such as, "I have 9 grapes and I eat 3 grapes. How many remain?" the student says "Nine ... eight, seven, six ... six!" This strategy is described as counting down from a number.

decimal

Used to describe aspects of the base-10 number system. The decimal point (. or ,) separates the whole number part of a number from its decimal part.

digit

A single symbol that is used to represent a number as a numeral. In the base-10 number system there are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

distributive property

In general, the distributive law (property) for multiplication over addition for real numbers states that for all real numbers a, b and c: $a(b + c) = a \times b + a \times c$.



division

For a finite set, the process of partitioning the set into subsets of equal size. For natural numbers, it expresses a given number as a multiple of a smaller number and any remainder.

division facts

Facts that draw on the inverse relationship between division and multiplication and are directly related to the multiplication facts, e.g. $2 \times 5 = 10$, so $\frac{10}{2} = 5$ and $\frac{10}{5} = 2$.

equal grouping

Dividing a collection, shape or object into a number of parts of equal shares.

equal sharing

Dividing a collection, shape or object into equal parts.

equivalence

Equal in value or meaning. Something such as an expression or statement that is essentially the same. Two or more sets that are capable of being mapped in a one-to-one relationship.

equivalent algebraic expressions

Expressions that are essentially the same, e.g. 3(x + 2) and 3x + 6 are equivalent expressions because the value of both the expressions remains the same for any value of x.

equation

A statement that includes the '=' symbol. Equations are used to show the equality of 2 expressions.

equivalent number sentences

Number sentences which have the same value, e.g. 527 + 96 = 527 + 100 - 4

estimation

The skill of conceptualising and mentally manipulating numbers or measurements to find an approximate answer. The capacity to make reasonable adjustments to estimates is essential in estimating.

expression

Two or more numbers or variables connected by operations.

factors

Let a, b and c be natural numbers such that $a \times b = c$, then *a* and *b* are factors (or divisors) of *c*, e.g. $3 \times 4 = 12$, so 3 and 4 are factors (divisors) of 12.

inverse operation

An operation in arithmetic which undoes the effect of another operation. Multiplication and division are inverse operations, as are addition and subtraction.

multiples

A multiple of a number is the product of that number and an integer. A multiple of a real number x is any number that is a product of x and an integer.

multiple-step problems

Problems which involve more than one calculation or process to solve them.



multiplication facts

The results associated with the products of pairs of natural numbers from 0 to 9, associated with reasons. They are foundational to arithmetic.

multiplicative

Problems or contexts that involve multiplication or division, e.g. calculating the number of seats in a theatre that has 30 rows of 24 seats.

natural numbers

The set $N = \{0, 1, 2, 3 ...\}$ or $N = \{1, 2, 3 ...\}$ depending on whether counting is started at 0 or 1. The elements of N are also called the counting numbers, used to count the number of elements in finite sets.

number sentence

A statement of equality or inequality using numbers, operations and common symbols,

e.g. 8 + 5 = 13 and $16 - \Box = 10$

numeral

The designation of a number in a given language, e.g. the number 'three' is designated by the Hindu-Arabic numeral 3, the Roman numeral III, and the Chinese numeral Ξ .

operation

The process of combining numbers or expressions. Operations are arithmetic – addition, subtraction, multiplication and division – and also include exponentiation and substitution.

order of operations

A set of conventions for evaluating expressions involving several operations. Operations in brackets are first, followed by exponents, multiplication/division, then addition/subtraction left to right.

partition numbers

Separating numbers additively or multiplicatively into 2 or more parts, e.g. 10 is 8 + 2, is 3 + 3 + 2; 12 divided into 6 equal parts of 2, $12 = 6 \times 2$, $12 \div 6 = 2$, $12 = 3 \times 4$, $12 \div 3 = 4$.

partitioning

The ability to think about numbers as made up of 2 or more parts. Numbers can be partitioned into standard or non-standard place value partitions such as 248 as 200 + 40 + 8 or 62 as 50 + 10 + 2.

product

The result of multiplying together 2 or more numbers or algebraic expressions.

quotient

The result of dividing one number or algebraic expression by another.

repeated addition

Adding the same number several times, e.g. $3 + 3 + 3 + 3 = 4 \times 3 = 12$. A strategy sometimes used for multiplication.

standard number sentences

Number sentences that are derived from semantic number sentences, i.e. how you think about a situation, e.g. 146 cm + ? = 160 cm, becomes ? = 160 - 146.



subitising

The capacity to visually recognise the size of a small set of objects without counting.

times by ten relationship

Each successive digit to the right of a number indicates a multiple of 10, e.g. in the number 2594 the 9 denotes 9 x 10 and the 5 denotes 50 x 10, and $326 = 10 \times 32.6$ or $326 = 100 \times 3.26$.